STUDENT'S NAME: ______________________________

TEACHER'S NAME: ______________________________

HURLSTONE AGRICULTURAL HIGH SCHOOL

2021

HIGHER SCHOOL CERTIFICATE ASSESSMENT TASK 4

Mathematics Advanced

General Instructions

- Preparation time – 10 minutes
- Working time – 3 hours
- Scanning and uploading time – 1 hour
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided in the Section I booklet
- In questions in Section II, show all relevant mathematical reasoning and/or calculations
- This examination paper is not to be removed from the examination centre

Total marks: 100

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10. The multiple choice answer sheet has been provided
- Allow about 15 minutes for this section

Section II – 90 marks (pages 13 – 37)

- Attempt Questions 11 – 16, writing your solutions in the spaces provided or on your own paper. There are 6 separate question/answer booklets.
- Allow about 2 hours and 45 minutes for this section.

Disclaimer: Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2021 HSC Mathematics Advanced Examination.

Section I

10 marks

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

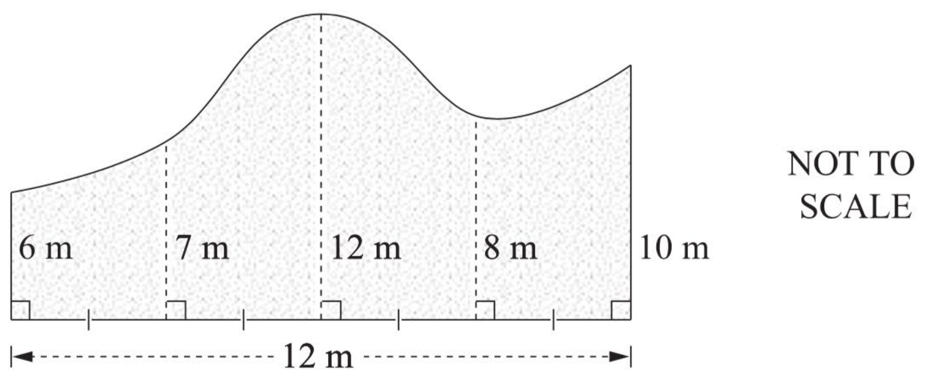
Use the multiple-choice answer sheet for Questions 1 – 10.

1. A pupil is asked to find the x-values of any possible points of inflection for the function $f(x) = 2x^3 + 12x^2 + 6x - 2$. What should his answer be?

A. -1 B. -2

C. 0 D. 1

2. The diagram below represents a field



What is the approximate area of the field, using four applications of the trapezoidal rule?

A. 105 m^2 B. 136 m^2

C. 210 m^2 D. 420 m^2

3. What is the value of $\int_{-3}^{2} |x+1|\,dx$?

A. $\dfrac{5}{2}$ B. $\dfrac{11}{2}$

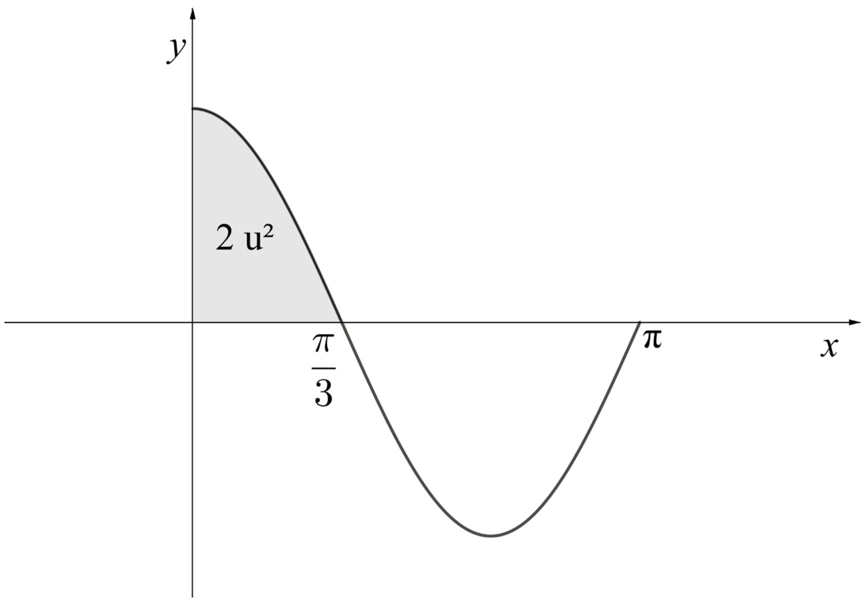
C. $\dfrac{13}{2}$ D. $\dfrac{17}{2}$

4. If $\tan\theta = \frac{2}{3}$ and θ is acute, what is the exact value of $\cos\theta$?

A. $\frac{2}{\sqrt{5}}$ B. $\frac{3}{\sqrt{5}}$

C. $\frac{3}{\sqrt{13}}$ D. $\frac{2}{\sqrt{13}}$

5. The diagram below shows the graph of $f(x) = a\cos bx$



The area of the shaded region is equal to 2 units^2.

What is the value of $\int_0^{\pi} f(x)\,dx$?

A. –4 B. –2

C. 2 D. 6

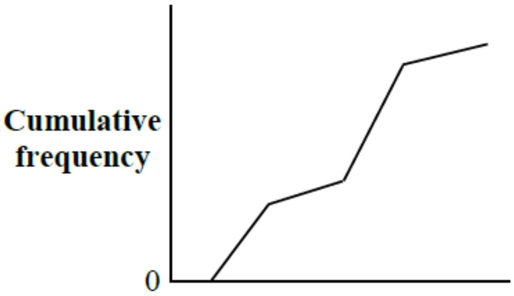
6. What is the natural domain of $f(x) = \frac{1}{e^x}$?

A. $(-\infty, \infty)$ B. $[0, \infty)$

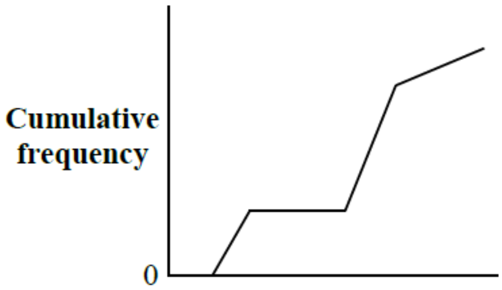
C. $(0, \infty)$ D. $(-\infty, 0]$

7. Which of the following CANNOT be a cumulative frequency polygon?

A.



B.



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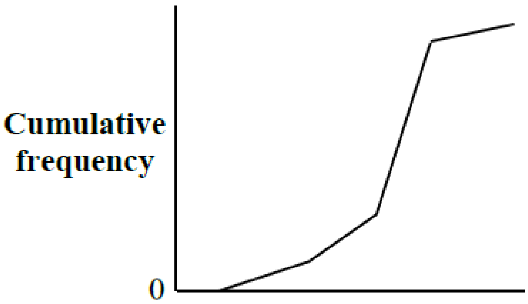
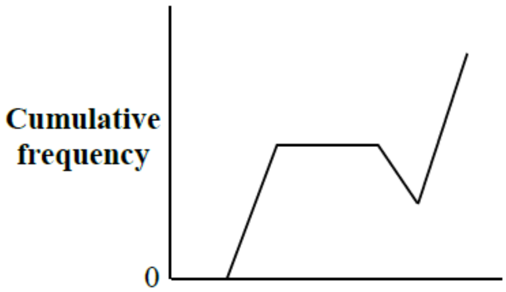
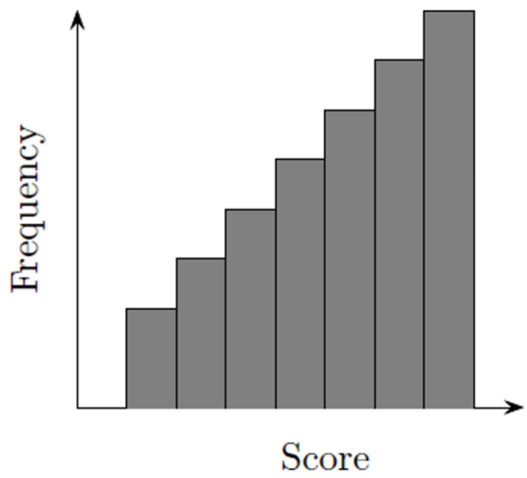


D.

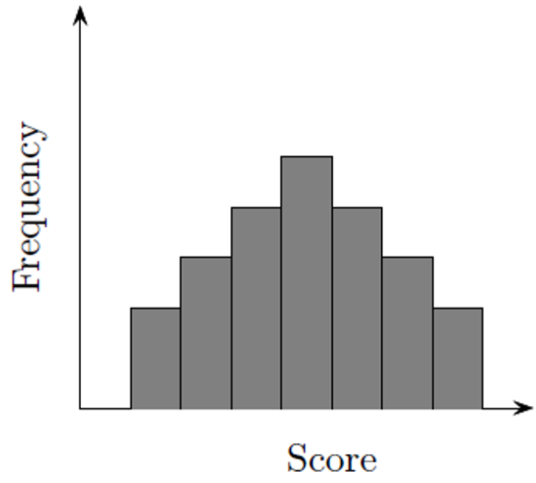


8. Which of the following graphs shows data with the largest standard deviation?

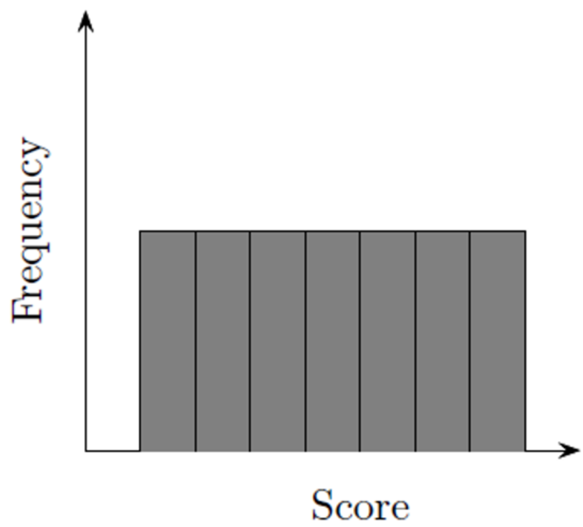
A.



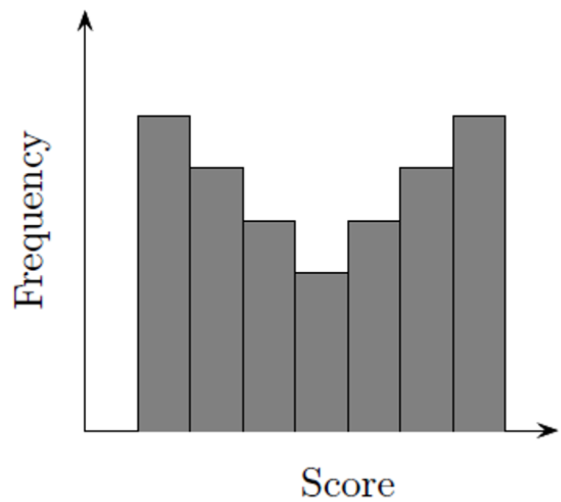
B.



C.



D.



9. A particular continuous random variable X has the following probability density function:

$$f(x) = \begin{cases} \dfrac{x}{32}, & 0 \le x \le 8 \\ 0, & \text{otherwise} \end{cases}$$

What is the median of this function?

A. $2\sqrt{2}$

B. 3.5

C. 4

D. $4\sqrt{2}$

10. The weight of chicken eggs is normally distributed with mean weight of 50 g and a standard deviation of 9 g. What percentage of eggs weigh between 41 g and 68 g?

A.	95%	B.	81.5%
C.	47.5%	D.	34%

End of Section I questions

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\dfrac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

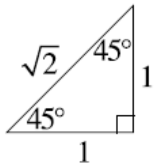
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$



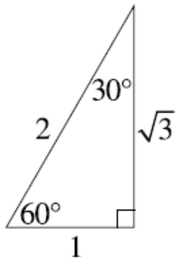
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

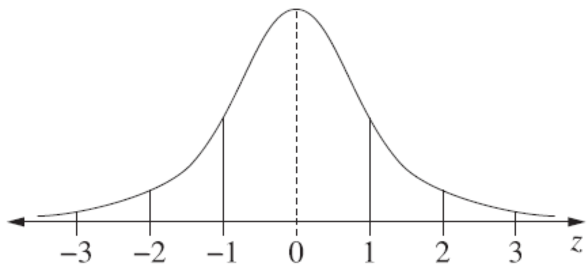
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x) dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x) dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x) dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx} dx = uv - \int v\frac{du}{dx} dx$$

$$\int_a^b f(x) dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

Hurlstone Agricultural High School

2021 Trial Higher School Certificate Examination
Mathematics Advanced

Name ______________________________ Teacher ______________________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

Section II **Name:** ________________________________

90 marks

Attempt Questions 11 – 16.

Allow about 2 hours and 45 minutes for this section.

Answer each question in the spaces provided. Extra working space is available after each question. If you need to use this extra space, you must clearly indicate this in the main solution space, and then clearly indicate the question number and part that you are answering in the extra space.

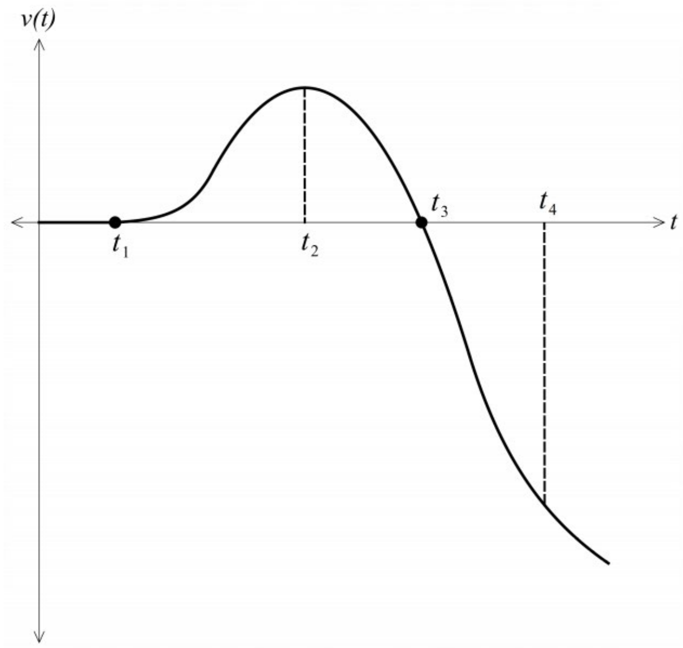
For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

2021 Mathematics Advanced Trial Examination Section II
Question 11 (15 marks)

Marks

(a) A particle moves in a straight line and is initially 10 metres right of the origin.

The velocity time graph shown below describes this motion



(i) What is the displacement of the particle at t_1 seconds? **1**

____

(ii) At what time/s is the particle at rest? **2**

____

(iii) At what time is the particle farthest to the right of the origin? **1**

____

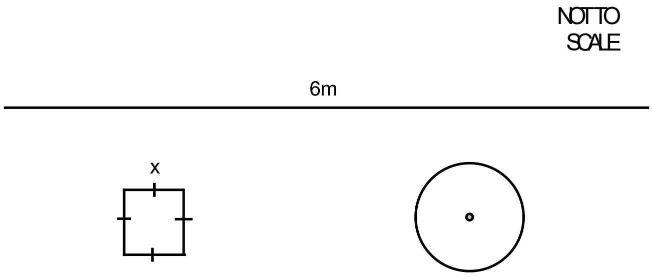
(b) For the curve $y = x^3 + 6x^2 + 9x$,

(i) Find any stationary points and determine their nature. **3**

(ii) Sketch the curve, showing all main features, including intercepts, stationary points and any points of inflection. **3**

(c) A piece of string of length 6 metres is cut into two pieces.

One piece forms a square with sides x cm and the other piece forms a circle.



(i) Show that the radius (r) of the circle in terms of x is given by **1**

$$r = \frac{3-2x}{\pi}.$$

(ii) Hence find the **lengths** of the two pieces of string which obtain the minimum area. **4**

Leave your answer in terms of π.

End of Question 11

Spare working space, Question 11.

Question 12 (15 marks) **Name:** ________________________________

Marks

(a) Evaluate $\int_1^3 x^{-2}\,dx$. **2**

(b) Find the area bounded by the x-axis and the curve $y = x^2 - 4$. **3**

(c) By firstly differentiating $y = \sqrt{2x^2 - 4}$, find $\int \frac{x}{\sqrt{2x^2 - 4}}\,dx$. **3**

(d) Consider the curves $y = x^3$ and $y = 7x^2 - 10x$, that intersect at three points.

(i) Show that two of these points of intersection are (0,0) and (2,8). **2**

(ii) Hence or otherwise, draw a sketch and calculate the area enclosed between the curves, between the two points found in (i) **5**

End of Question 12

Spare working space, Question 12.

Question 13 (15 marks) **Name:** ______________________

Marks

(a) Show that $\dfrac{\sec\theta - \sec\theta\cos^4\theta}{1+\cos^2\theta} = \sin\theta\tan\theta$ **2**

(b) Solve $\sin\left(x+\dfrac{\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$ for $0 \le x \le 2\pi$ **2**

(c) The curve $y = f(x)$ passes through the point (0,7).

If its gradient function is given by $\dfrac{dy}{dx} = 1 - 6\sin 3x$, find the equation of the curve. **2**

(d) A particle moves in a straight line.

At time t seconds, its distance x metres from a fixed point 0 on the line

is given by $x = 1 - \cos 2t$.

(i) Sketch the graph of x as a function of t for $0 \leq t \leq \pi$ **3**

(ii) Using your graph, or otherwise, find the times when the particle is at rest and the position of the particle at these times. **2**

(e) (i) Differentiate $\sin^2 x$ **1**

(ii) Hence, calculate $\int_0^{\frac{\pi}{4}} (\sin x + \cos x)^2 \, dx$, leaving your answer in exact form. **3**

End of Question 13

Spare working space, Question 13.

Question 14 (15 marks) **Name:** ______________________________

Marks

(a) Find derivatives for the following, with respect to x.

(i) $\ln\left(x^2+2\right)$ **1**

(ii) $3^x e^x$ **2**

(b) Show that the curve $y=2x^2-\ln\left(\dfrac{x}{2}\right)-4$ has a stationary point at $\left(\dfrac{1}{2},\ln 4-3\dfrac{1}{2}\right)$. **3**

(c) (i) Find the co-ordinates of the point of intersection of the line $y = 3$ and the curve $y = e^x + 1$. **1**

(ii) Draw a neat sketch of the area bounded by $y = 3$, the y-axis and the curve $y = e^x + 1$. **1**

(iii) Calculate the exact area drawn in part (ii). **2**

(d) The acceleration of a particle, P, in m s $^{-2}$ is $\dfrac{d^2x}{dt^2} = e^{-t} + e^{-2t}$ where t is measured in seconds.

Initially, the displacement of the particle is $x = \dfrac{3}{4}$ m, travelling at a velocity $\dfrac{dx}{dt} = -\dfrac{3}{2}$ m s $^{-1}$.

(i) Show that the displacement of the particle is given by:

$x = e^{-t} + \dfrac{1}{4}e^{-2t} - \dfrac{1}{2}$. **2**

(ii) Find the limit of the displacement of P, and hence the limit of the distance that P travels after $t = 0$. **1**

(e) Find the value of k such that $\int_{-2}^{0} \frac{x^2}{x^3 - 2}\, dx = \ln k$ **2**

End of Question 14

Spare working space, Question 14.

Question 15 (15 marks) **Name:** ____________________

Marks

(a) The average monthly relative humidity (in %) of city A is shown in the stem-and-leaf plot.

Stem	**Leaf**			
6	1	1	1	2
7	3	5	8	
8	3	7	7	

(i) Find the median and the inter-quartile range. **2**

____

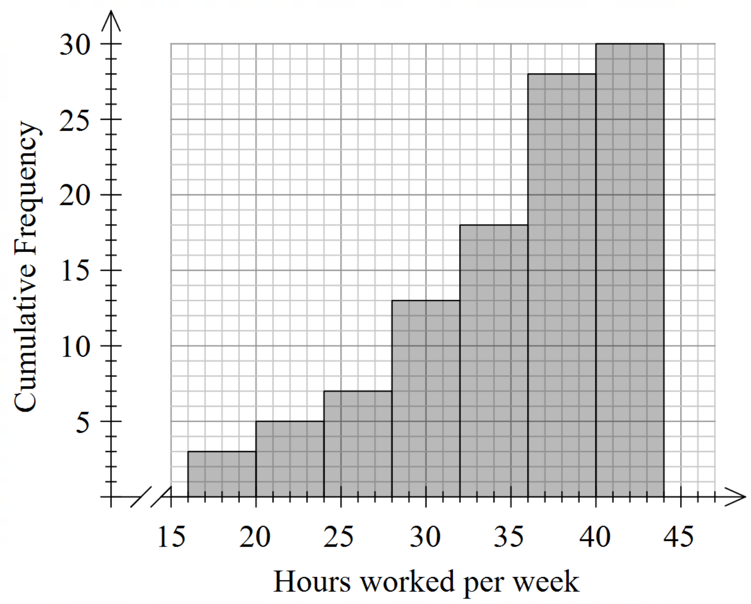
____

(ii) Draw a box-and-whisker plot to represent the data. **2**

(b) Cole is designing a survey to ask his co-workers about their job satisfaction.

One of Cole's survey questions asked how many hours each respondent works at the company.

The results are shown in the cumulative frequency histogram below.



(i) What is the range of responses that gave the greatest 40% of hours worked? **1**

(ii) Use the classes in the cumulative frequency histogram to estimate the mean hours worked by the respondents surveyed. **3**

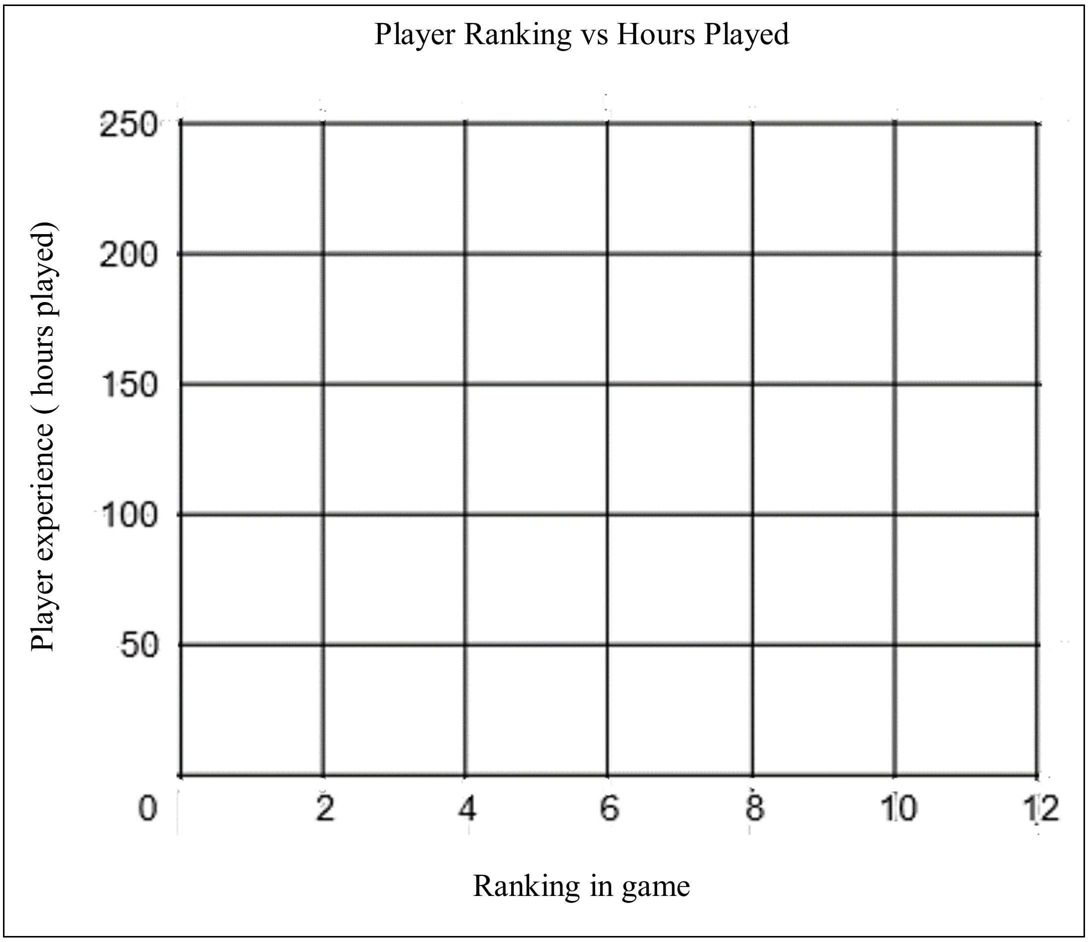
(c) Ten students were ranked on their computer gaming ability on a new game.

Each student also calculated the number of hours that they have played the game.

The results are recorded in the table below.

Rank	1	2	3	4	5	6	7	8	9	10
Hours Played	198	143	88	102	82	94	54	36	20	12

(i) Using the axes below draw the scatterplot for the data in the table. **1**



(ii) Calculate, to 2 decimal places, Pearson's correlation coefficient, *r,* and describe the relationship between a player's rank and the number of hours that they have played the game. **2**

____

____

____

(iii) Find the equation of the least-squares regression line for the data given above.

Give your answer correct to 2 decimal places. **2**

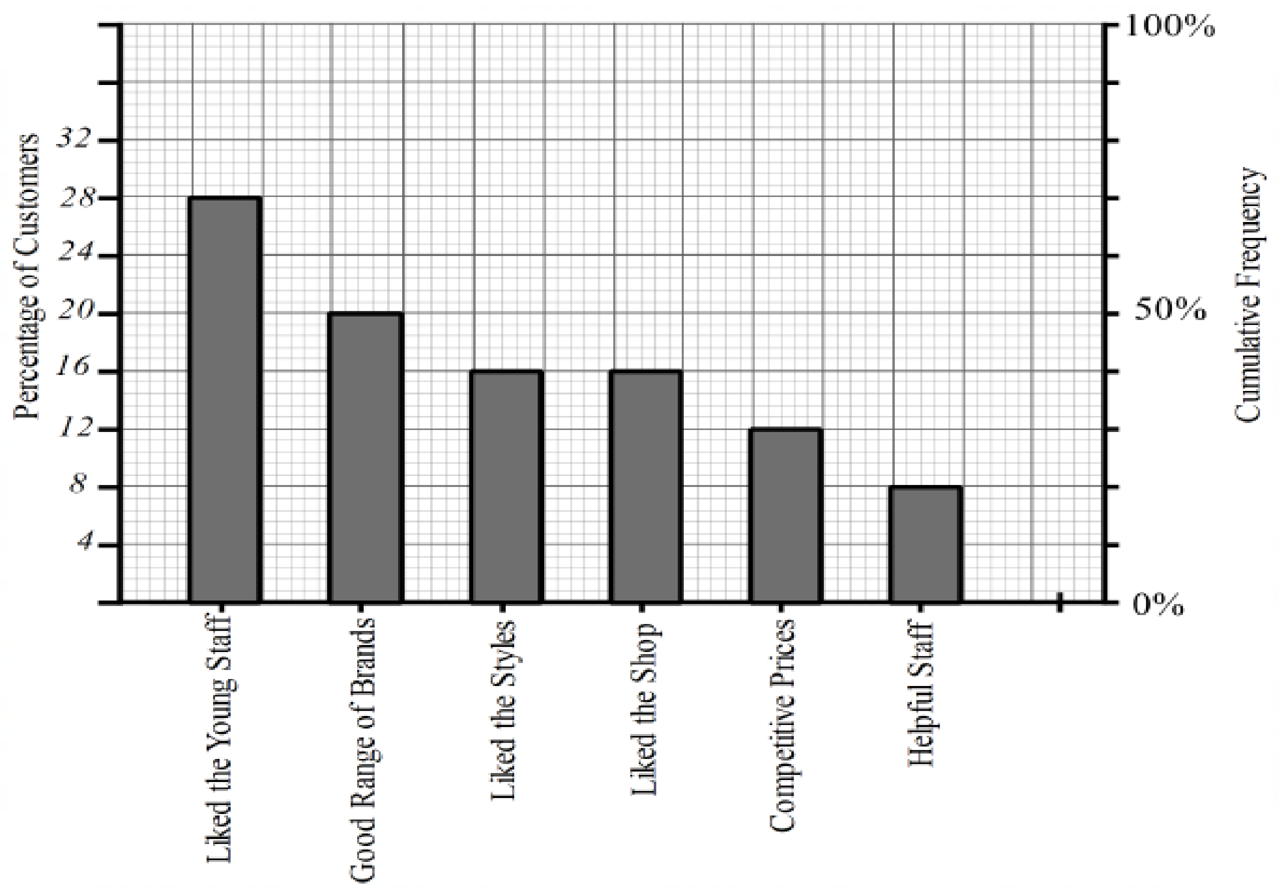
____

____

____

____

(d) The chart below shows the reasons that 25 customers gave for shopping at a local clothing store.



Draw the Pareto line on the chart above. **2**

End of Question 15

Spare working space, Question 15.

2021 Mathematics Advanced Trial Examination Section II
Question 16 (15 marks) **Name:** ______________________________

Marks

(a) (i) Show that the function $f(x)=\frac{\pi}{12}\sin\left(\frac{\pi x}{6}\right),[0,6]$

is a probability density function. **3**

(ii) For a particular continuous random variable *X*, find $P(X\le 4)$
for the function described in (i) **2**

(b) A cumulative distribution function is given by $F(x)=\frac{x^3-8}{335}$.
Find the interquartile range of the continuous probability distribution. **2**

(c) A number of fish species are subject to minimum length regulations when they are caught.

ie. fish shorter than a given length must be returned to the water if caught.

Two such species are Red Snapper and Barramundi which have minimum lengths of 30 cm and 55 cm respectively.

A fishing tour operator in Northern Australia has made observations over a long period of time and found that, when measured in cm, both the variables 'R' (the lengths of caught Red Snapper) and 'B' (the lengths of caught Barramundi), are normally distributed.
'R' has a mean of 36 cm and standard deviation of 3 cm. 'B' has a standard deviation of 4 cm.

(i) Calculate the mean length of Barramundi caught if 2.5% of Barramundi caught are less than 54 cm. **2**

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____

(ii) Calculate the *z*-scores for the minimum allowed lengths for both variables, R and B, and comment upon what this means in terms of which of the two species are more likely to be returned to the water after capture. **3**

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(d) Historical data for a particular aptitude test show that its completion time has a mean of 5 minutes with a standard deviation of 30 seconds.

As part of the selection process for an available job, an employer requires candidates to complete the test faster than 90% of all applicants to progress to the next stage.

An extract from a probability table for the standard normal distribution is shown below.

	second decimal place									
z	+ .00	+ .01	+ .02	+ .03	+ .04	+ .05	+ .06	+ .07	+ .08	+ .09
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441

Darcy completed the aptitude test in 4 minutes and 23 seconds.

Did Darcy qualify for the next stage of selection? Justify your answer by demonstrating your knowledge of the normal distribution and the application of z-scores to the problem.

3

End of Question 16

End of Examination.

Hurlstone Agricultural High School

2021 Trial Higher School Certificate Examination
Mathematics Advanced

Name _____**SOLUTIONS**__________ Teacher _______________

Section I – Multiple Choice Answer Sheet

	A	B	C	D
1.	○	●	○	○
2.	●	○	○	○
3.	○	○	●	○
4.	○	○	●	○
5.	○	●	○	○
6.	●	○	○	○
7.	○	○	○	●
8.	○	○	○	●
9.	○	○	○	●
10.	○	●	○	○

Solutions:

Q1

$$f(x) = 2x^3 + 12x^2 + 6x - 2$$
$$f'(x) = 6x^2 + 24x + 6$$
$$f''(x) = 12x + 24$$

For point of inflection,

$f''(x) = 0, \; x = -2 \quad Ans.\ B$

Q2 Apply Trapezoidal Rule

$$A = \frac{12 \div 4}{2}\{6 + 10 + 2(7 + 12 + 8)\}$$
$$= \frac{3}{2}(70) = 105 \quad Ans\ A$$

Q2 An alternate solution suggestion:

A) 105 m^2

Suggested solution

$12 \times 10 = 120$ m^2 which is an over estimation. Thus Option A.

Q3

C) $\frac{13}{2}$

Suggested Solution

Integral of $\int_{-3}^{2}|x+1|\,dx$ is equal to the area formed by the triangles between [-3, 2], the curve and *x*-axis.

So □ $\int_{-3}^{2}|x+1|\,dx = \frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 3 \times 3 = \frac{13}{2}$

Q4

The solution can be found using SOHCAHTOA in a right triangle, with adjacent side 3 and hypotenuse $\sqrt{13}$ using Pythagoras. Answer **C**

Q5

The area below the *x*-axis has area 4 square units, so the integral is 2 −4= −2. Answer **B**

Q6

e^x is always greater than zero, so there is nowhere that the function won't exist. Answer : **A**

Q7

Answer: D . Cumulative frequency is never going to be a decreasing function.

Q8

Answer: **D** due to a lot of low scores and high scores the distance between the mean and each of the individual scores will be greater. Hence greater Standard deviation.

Q9

We find the value of X for which the integral will equal ½

$\int_0^k \frac{x}{32}dx = \left[\frac{x^2}{64}\right]_0^k = \frac{k^2}{64} \rightarrow k^2 = 32,\ k = 4\sqrt{2}$ (k must be positive on $\frac{x}{32}$ Answer **D**

Q10

Values range from 1 standard deviation below to 2 standard deviations above the mean.

From a normal distribution, we have approx. 34% of scores below the mean and 47·5% of scores above the mean. Total will be 81·5%

Answer **B**

Answers for MC: (1) B, (2) A, (3), (4) , (5), (6), (7), (8)

Q1

$f(x)=2x^3+12x^2+6x-2$

$f'(x)=6x^2+24x+6$

$f''(x)=12x+24$

For point of inflection,

$f''(x)=0,\ x=-2 \quad Ans.\ B$

Q2 Apply Trapezoidal Rule

$$A=\frac{12\div 4}{2}\{6+10+2(7+12+8)\}$$

$$=\frac{3}{2}(70)=105 \quad Ans\ A$$

Question No. 11 Solutions and Marking Guidelines

Outcomes Addressed in this Question

MA 12-6 Applies appropriate differentiation methods to solve problems

Outcome	**Solutions**	**Marking Guidelines**
MA 12-6	a) i) 10m to the right of origin. Accept 10 m ii) $0 < t < t_1$ *and* t_3 iii) t_3	i) 1 mark – correct ii) 2 marks for correct 1 mark for one correct iii)1 mark –correct

b) i) $y = x^3 + 6x^2 + 9x$

$$\frac{dy}{dx} = 3x^2 + 12x + 9$$
$$= 3(x^2 + 4x + 3)$$
$$= 3(x+3)(x+1)$$
$$\frac{d^2y}{dx^2} = 6x + 12$$

Stationary points occur when $\frac{dy}{dx} = 0$

$3(x+3)(x+1) = 0$

$x = -3$ *or* $x = -1$

When $x = -3$, $\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} = 6(-3) + 12 = -6 < 0$ max at $(-3, 0)$

$x = -1$, $\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} = 6(-1) + 12 = 6 > 0$ min at $(-1, -4)$

Part (b)(i)
3 marks for correct solution

2 marks – obtain max and min pts correct

1 mark – some progress

b) ii)

$\frac{d^2y}{dx^2} = 0$, then $6x + 12 = 0$, $x = -2$, $y = (-2)^3 + 6(-2)^2 + 9(-2) = -2$

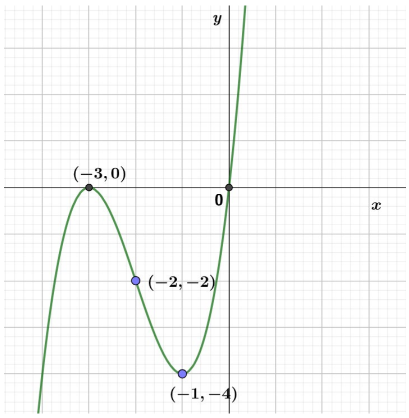
Point of inflection at $(-2, -2)$

Testing

x	-3	-2	-1
$\frac{d^2y}{dx^2}$	-6	0	6

Concavity changes

There is a point of inflection at $(-2, -2)$



Part (b)(ii)
3 marks for correct solution

1 mark for correct finding of (-2,-2)
1 mark for testing pt of inflection
1 mark for correct graph

C) i) Total Perimeter:

Part (c)

	$4x + 2\pi r = 6$ $2\pi r = 6 - 4x$ $r = \dfrac{6-4x}{2\pi}$ $= \dfrac{3-2x}{\pi}$ C) ii) Total Area $= A = \pi r^2 + x^2$ $= \pi\left(\dfrac{3-2x}{\pi}\right)^2 + x^2$ $= \dfrac{(3-2x)^2}{\pi} + x^2$ $\dfrac{dA}{dx} = \left(\dfrac{1}{\pi}\right)2(3-2x)(-2) + 2x$ $= \dfrac{-12 + 8x + 2\pi x}{\pi}$ $\dfrac{d^2A}{dx^2} = \dfrac{8+2\pi}{\pi} > 0 \Rightarrow$ minimum Minimum area occurs when $\dfrac{dA}{dx} = 0$ $\therefore -12 + 8x + 2\pi x = 0$ $x(8+2\pi) = 12$ $x = \dfrac{12}{8+2\pi} = \dfrac{6}{4+\pi}$ Length required for square is $= 4 \times \dfrac{6}{4+\pi}$ $= \dfrac{24}{4+\pi}$ Length required for circle is $6 - \dfrac{24}{4+\pi} = \dfrac{6\pi}{4+\pi}$	i) 1 mark – correctly shown ii) 4 marks for correct solution 1 mark for correct finding of area 1 mark for showing min area 1 mark for correct value of x 1 mark for correct length of square and circle 1 mark – some progress

<table>
<tr><td colspan="3">Year 12   Mathematics Advanced   Task 4   Examination 2021
Question No.12   Solutions and Marking Guidelines</td></tr>
<tr><td colspan="3">MA 12-7: Applies the concepts and techniques of indefinite and definite integrals in the solutions of problems.</td></tr>
<tr><th>Part</th><th>Solutions</th><th>Marking Guidelines</th></tr>
<tr><td>(a)</td><td>

$$\int_1^3 x^{-2}\,dx = \left[-x^{-1}\right]_1^3$$
$$= -3^{-1} - \left(-1^{-1}\right)$$
$$= \frac{2}{3}$$

</td><td>2 marks
Correct Solution

1 mark
Single error</td></tr>
<tr><td>(b)</td><td>Roots of quadratic are -2 and 2
Using the symmetry of an even function we have:

$$A = \left|2\int_0^2 (x^2 - 4)\,dx\right|$$
$$= 2\left[\frac{1}{3}x^3 - 4x\right]_0^2$$
$$= 2\left|\frac{1}{3}(2)^3 - 4(2) - 0\right|$$
$$= 10\frac{2}{3}$$

</td><td>3 marks
Correct solution

2 marks
Single error

1 mark
Substantial progress that would lead to a correct answer</td></tr>
<tr><td>(c)</td><td>

$$\frac{d}{dx}\left(\sqrt{2x^2 - 4}\right) = \frac{d}{dx}\left(\left(2x^2 - 4\right)^{\frac{1}{2}}\right)$$
$$= \left(\frac{1}{2}\right)(4x)\left(2x^2 - 4\right)^{-\frac{1}{2}}$$
$$= \frac{2x^2}{\sqrt{2x^2 - 4}}$$

Hence, $\displaystyle\int \frac{x}{\sqrt{2x^2 - 4}}dx = \frac{1}{2}\sqrt{2x^2 - 4} + c$

</td><td>3 marks
Correct solution

2 marks
Single error

1 mark
Substantial progress that would lead to a correct answer</td></tr>
</table>

(d) i

Equating the two equations and solving for x we have:

$$x^3 = 7x^2 - 10x$$
$$x^3 - 7x^2 + 10x = 0$$
$$x(x^2 - 7x + 10x) = 0$$
$$x(x-2)(x-5) = 0$$
$$x = 0,2,5$$

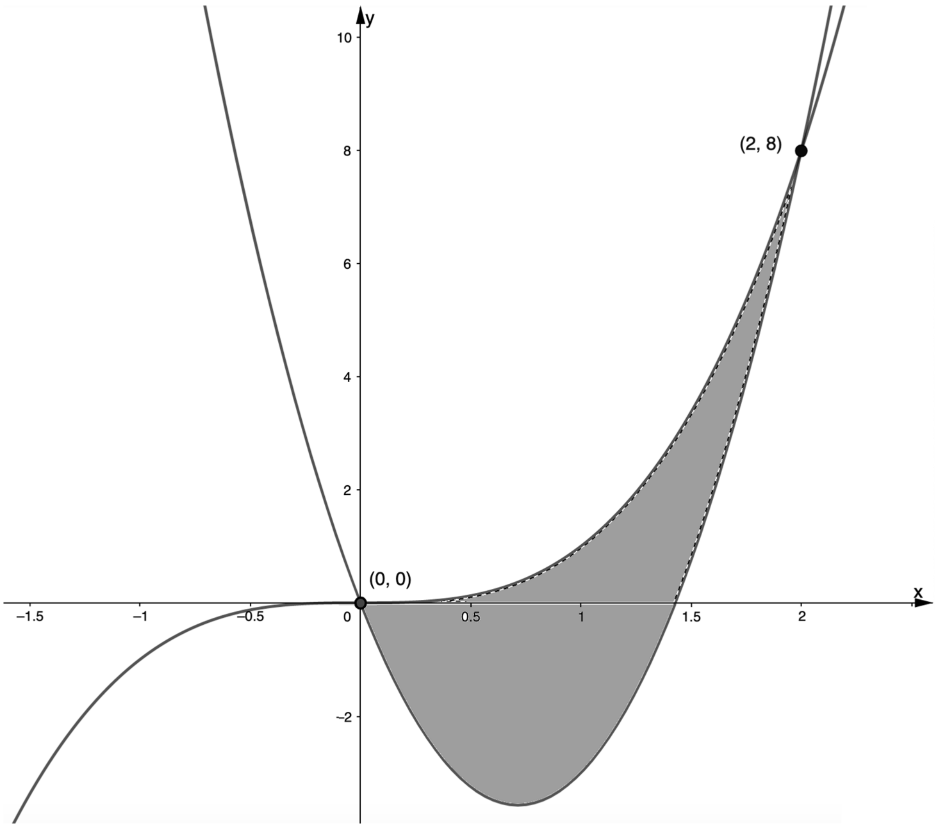
$$y(0) = 0\ ;\ y(2) = 2^3 = 8$$

Hence (0,0) and (2,8) are coordinates of intersection

2 marks
Correct solution

1 mark
Single Error

ii



$$A = \int_0^2 \left(x^3 - \left(7x^2 - 10x\right)\right)dx$$
$$= \left[\frac{1}{4}x^4 - \frac{7}{3}x^3 + 5x^2\right]_0^2$$
$$= \frac{1}{4}(2)^4 - \frac{7}{3}(2)^3 + 5(2)^2 - 0$$
$$= 5\frac{1}{3}$$

5 marks
Correct solution

4 marks
Single error

3 marks
Substantially correct solution including correct graph

2 marks
Substantial progress that would lead to a correct answer

1 mark
Minimal progress that would lead to a correct answer

MC Solutions

Q2

A) 105 m^2

Suggested solution

$12 \times 10 = 120$ m^2 which is an over estimation. Thus Option A.

Q3

C) $\frac{13}{2}$

Suggested Solution

Integral of $\int_{-3}^{2} |x+1| \, dx$ is equal to the area formed by the triangles between [-3, 2], the curve and *x*-axis.

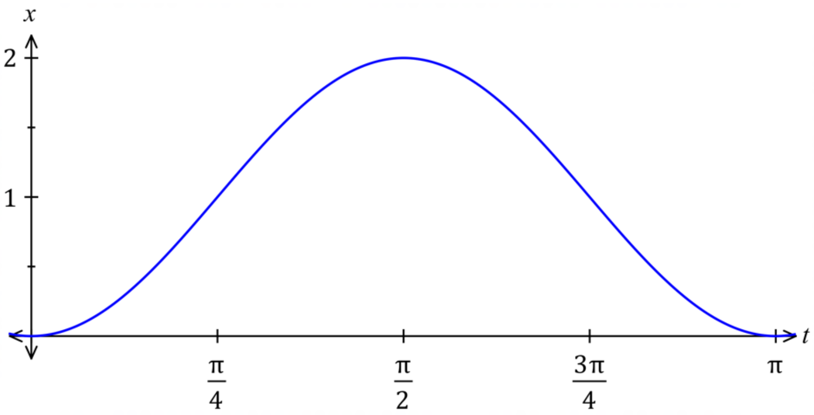
So □ $\int_{-3}^{2} |x+1| \, dx = \frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 3 \times 3 = \frac{13}{2}$

Year 12	Mathematics Advanced 2021	TASK 4
Question No. 13	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

MA 12-5: Applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs.

Part / Outcome	Solutions	Marking Guidelines
(a)	$\text{LHS} = \dfrac{\sec\theta\left(1-\cos^4\theta\right)}{1+\cos^2\theta}$ $= \dfrac{1}{\cos\theta} \times \dfrac{\left(1-\cos^2\theta\right)\left(1+\cos^2\theta\right)}{1+\cos^2\theta}$ $= \dfrac{1}{\cos\theta} \times \left(1-\cos^2\theta\right)$ *show* this $= \dfrac{1}{\cos\theta} \times \sin^2\theta$ *need* this $= \dfrac{\sin\theta}{\cos\theta} \times \sin\theta$ to *show* this $= \sin\theta\tan\theta$	**2 marks** – Correct solution **1 mark** – Substantially correct
(b)	$\sin\left(x+\dfrac{\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$ acute related angle $x+\dfrac{\pi}{6} = \dfrac{\pi}{3}$ $= \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$ 3rd, 4th *quadrant* $x = \dfrac{7\pi}{6}, \dfrac{3\pi}{2}$	**2 marks** – Correct solution **1 mark** – Substantially correct (finds acute related angle, or equivalent merit) Also note that answering in degrees gives number outside the domain – You must answer in radians
(c)	$\dfrac{dy}{dx} = 1-6\sin 3x$ $f(x) = x+\dfrac{6\cos 3x}{3}+C$ $f\left(0\right) = 0+\dfrac{6\cos 0}{3}+C = 7$ $7 = 2+C \quad \therefore C = 5$ $f(x) = x+\dfrac{6\cos 3x}{3}+5$ $f\left(x\right) = x+2\cos 3x+5$	**2 marks** – Correct solution **1 mark** – Substantially correct

	Question 13 continued…	
(d)(i)	$x = 1 - \cos 2t, \quad 0 \le t \le \pi$ $\Rightarrow$ amplitude $= 1$, period $= \pi$ 	**3 marks** – Correct solution (shape of curve is important here… gradient is zero at $x = 0$, π… <u>this needs to be shown clearly</u>) **2 marks** – Substantially correct (axes, domain, range labelled correctly) **1 mark** – Partial progress towards correct solution
(d)(ii)	Particle is at rest when $v = 0$ ie $\dfrac{dx}{dt} = 0$ (stationary points on graph) $t = 0, \dfrac{\pi}{2}, \pi$ position at these times is $x = 0,\ 2, 0$	**2 marks** – Correct solution (either determined correctly from drawn graph, or justified algebraically from given function. Also note that stating coordinates, like (π, 0) is not answering the <u>actual</u> question of when and where) **1 mark** – Substantially correct
(e)(i)	$y = \sin^2 x$ $\dfrac{dy}{dx} = 2\cos x \sin x \quad (= \sin 2x)$	**1 mark** – correct solution
(e)(ii)	$\int_0^{\frac{\pi}{4}} (\sin x + \cos x)^2 \, dx$ $= \int_0^{\frac{\pi}{4}} \left(\sin^2 x + 2\sin x \cos x + \cos^2 x\right) dx$ $= \int_0^{\frac{\pi}{4}} \left(1 + 2\sin x \cos x\right) dx \quad \left(= \int_0^{\frac{\pi}{4}} \left(1 + \sin 2x\right) dx\right)$ $= \left[x + \sin^2 x\right]_0^{\frac{\pi}{4}}$ (from (i)) $\leftarrow$ *must* be used $= \left[\dfrac{\pi}{4} + \left(\dfrac{1}{\sqrt{2}}\right)^2\right] - 0$ $= \dfrac{\pi}{4} + \dfrac{1}{2}$	**3 marks** – Correct solution **2 marks** – Substantially correct **1 mark** – Partial progress towards correct solution (must get to line 3 for 1 mark – ie expand and simplify integral) Also note the word "Hence" without the phrase "or otherwise". You MUST use your result in part (i) for full marks

	Just FYI, the three most common issues with this question are highlighted by the three most common comments I wrote in my responses, which are below (*many* various versions of the (d)(ii) comment were used) This is here as a reminder that DETAIL is important. And detail is often where the marks are. Whether it's reading the detail in the question, or paying attention to detail in your solutions (b) domain is $0 < x < 2\pi$ ie $0 < x < 6.28$ Working in degrees, your answers/values are outside this domain. If converting to degrees for your working, you MUST also convert back to radians (d)(ii) The shape of the curve is vital in this question! Your graph clearly shows no horizontal gradient at pi (zero is on the edge), so that can not be considered as stationary. Had you demonstrated that you obtained these results algebraically, the 2nd mark would have been awarded (e) Hence!!! (and no 'otherwise') ---> you must use part (i)	

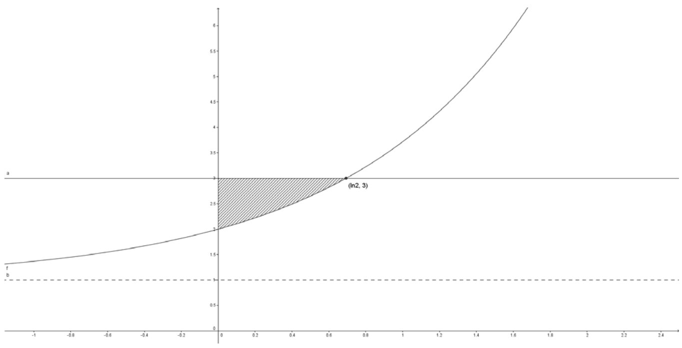
Year 12	Mathematics Advanced 2021	TASK 4
Question No. 14	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

MA 12-6: Applies appropriate differentiation methods to solve problems
MA 12-7: Applies the concepts and techniques of indefinite and definite integrals in the solutions of problems..

<table>
<tr><th>Part / Outcome</th><th>Solutions</th><th>Marking Guidelines</th></tr>
<tr><td>MA 12-6
(a)</td><td>(i) $\dfrac{2x}{x^2+2}$
(ii)
$$\frac{d}{dx}e^{x\ln 3}e^x = e^{x\ln 3}e^x + e^x\ln 3\, e^{x\ln 3}$$
$$=3^x e^x + 3^x e^x \ln 3$$
$$=3^x e^x\left(1+\ln 3\right)$$</td><td>(a)(i) 1 mark: Correct answer

(ii) 2 marks: Correct solution including 3^x in solution.
1 mark: Correct substitution of product rule.</td></tr>
<tr><td>(b)</td><td>$y' = 4x - \dfrac{1}{x}$ Either by solving $y'=0$ or substituting $x=\dfrac{1}{2}$ show that stationary point exists.
By substitution, show that $y\left(\dfrac{1}{2}\right)=\ln 4 - 3\dfrac{1}{2}$ showing utilisation of log law $-\ln\left(\dfrac{1}{4}\right)=+\ln 4$
Therefore, $\left(\dfrac{1}{2}, \ln 4 - 3\dfrac{1}{2}\right)$ is a stationary point.</td><td>(b) 3 marks: Correct solution clearly communicated.
2 marks: Substantially correct.
1 mark: Partial relevant progress towards correct solution</td></tr>
<tr><td>MA 12-7
(c)</td><td>(i) $\left(\ln 2, 3\right)$

(ii)

(iii) Area is equal to the integral between the two functions.
$$\int_0^{\ln 2} 3-\left(e^x+1\right)dx = \int_0^{\ln 2} 2-e^x\,dx$$
$$=\left[2x-e^x\right]_0^{\ln 2}$$
$$=\left(2\ln 2-2\right)-\left(0-1\right)$$
$$=2\ln 2-1$$</td><td>(c)(i) 1 mark: Correct x value.

(ii) 1 mark: Correct line, curve and shading.

(iii) 2 marks – Correct solution from previous (i) (ii)
1 mark: Correct integral statement.
1 mark: Equivalent correct answer from slightly incorrect integral.</td></tr>
</table>

(d)	(i) $\frac{dx}{dt} = -e^{-t} - \frac{1}{2}e^{-2t} + c_1$ Initial conditions: $-\frac{3}{2} = -e^{0} - \frac{1}{2}e^{0} + c_1 \quad \rightarrow c_1 = 0$ $\therefore \frac{dx}{dt} = -e^{-t} - \frac{1}{2}e^{-2t}$ $x = e^{-t} + \frac{1}{4}e^{-2t} + c_2$ Initial conditions: $\frac{3}{4} = e^{0} + \frac{1}{4}e^{0} + c_2 \quad \rightarrow c_2 = -\frac{1}{2}$ $\therefore x = e^{-t} + \frac{1}{4}e^{-2t} - \frac{1}{2}$ as required.	**(d)(i) 2 marks:** Correct solution including testing initial conditions for constants for both primitives. **1 mark:** Correct solution for one of the primitives.
(d)(ii)	(ii) $\lim_{t \to \infty} x = 0 + \frac{1}{4}(0) - \frac{1}{2} = -\frac{1}{2}$ So, the limiting change in displacement from the starting point is 1·25m.	**(ii) 1 mark:** Correct answer. Accept correct numerical expression.
(e)	(i) $\ln k = \frac{1}{3}\int_{-2}^{0} \frac{3x^2}{x^3 - 2}\,dx$ $= \frac{1}{3}\left[\ln\left\|x^3 - 2\right\|\right]_{-2}^{0}$ $= \frac{1}{3}(\ln 2 - \ln 10)$ $= \frac{1}{3}\ln\left(\frac{1}{5}\right)$ $\therefore k = \sqrt[3]{\frac{1}{5}}$	**(e) 2 marks** – Correct solution. **1 mark** – Correct primitive statement with boundaries, or correct simplification of log law to find *k*.

Multiple Choice:

6. What is the natural domain of $f(x)=\frac{1}{e^x}$?

A. $(-\infty,\infty)$

B. $[0,\infty)$

C. $(0,\infty)$

D. $(-\infty,0]$

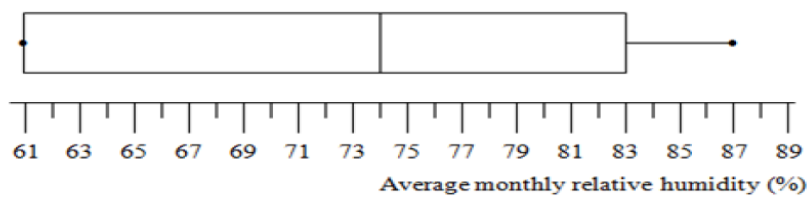
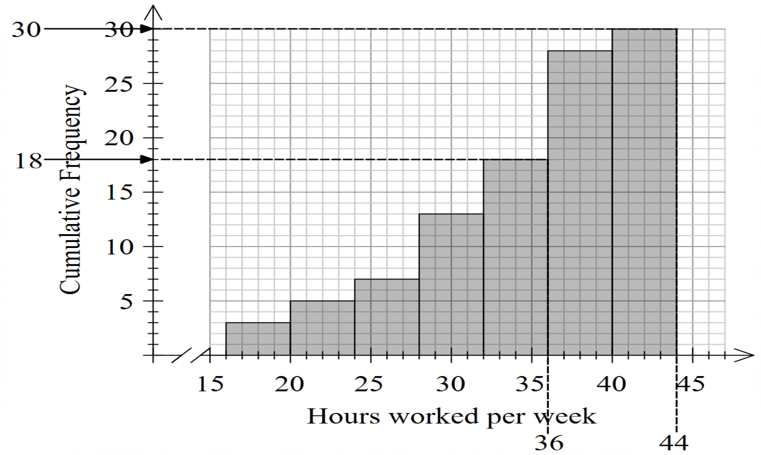
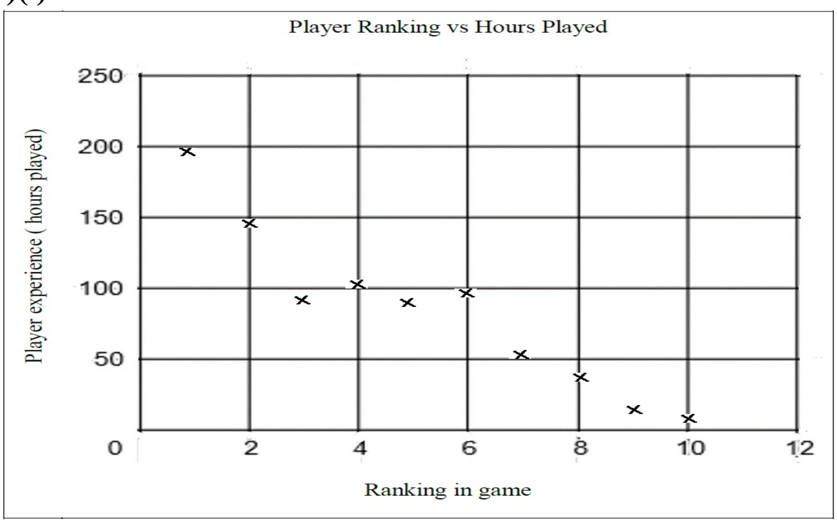
---

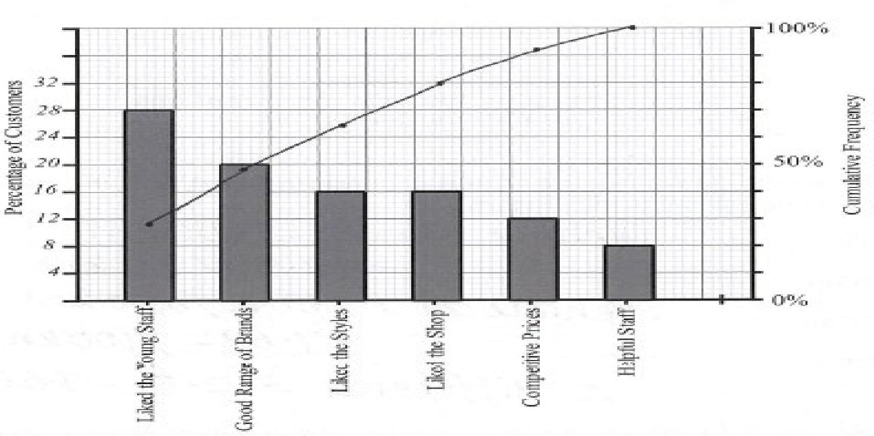
e^x is always greater than zero, so there is nowhere that the function won't exist. Answer : **A**

Question No. 15 Solutions and Marking Guidelines

Outcomes Addressed in this Question

MA 12-8: Solves problems using appropriate statistical processes.

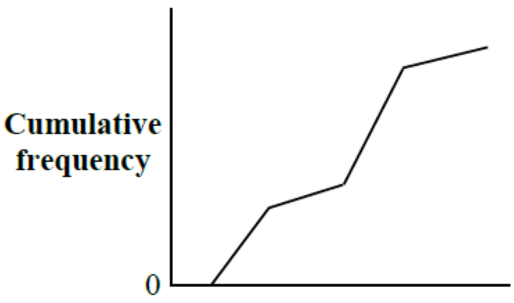
Outcome	Solutions	Marking Guidelines
MA 12-8	**(a)(i)** Median $=\left(\dfrac{73+75}{2}\right)\%=74\%,\ \ Q_1=61\%, Q3=83\%$ $\therefore$ Inter-quartile range $=(83-61)\%=22\%$	**Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the solution
	(ii) 	**Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the solution
MA 12-8	**(b)(i)** $40\% \times 30 = 12$  Top 12 respondents are from 18 to 30, which are the top two classes. These respondents worked 36 to 44 hours per week.	**Award 1** mark for the correct solution.
	(ii) Class centres are the middle of each bar of the histogram. The number of respondents in each class is the change in height for each bar of the histogram. Sum of scores: $18\times3+22\times2+26\times2+30\times6+34\times5+38\times10+42\times2=964$ Mean: $\dfrac{964}{30}=32.1$	**Award 3** marks for the correct solution. **Award 2** mark for substantial progress towards the correct solution. **Award 1** mark for some progress towards the correct solution.
MA 12-8	**(c)(i)** 	**Award 1** mark for the correct location of data on the graph

MA 12-8	**(ii)** $r = -0.94$ There is a very strong negative relationship between rank and gaming hours. As time played increases, rank decreases. (iii) $a = 180.47$ $b = -17.74$ $y = -17.74x + 180.47$ **(d)**  **Steps for Calculating Cumulative Frequency to draw the Pareto Line.** 1) $28\% \Rightarrow \text{Cumulative Frequency} = 28\%$ 2) $20\% \Rightarrow \text{Cumulative Frequency} = 48\%$ 3) $16\% \Rightarrow \text{Cumulative Frequency} = 64\%$ 4) $16\% \Rightarrow \text{Cumulative Frequency} = 80\%$ 5) $12\% \Rightarrow \text{Cumulative Frequency} = 92\%$ 6) $8\% \Rightarrow \text{Cumulative Frequency} = 100\%$	**Award 2** marks for correct calculation of *r* with correct description **Award 1** mark for correct calculation of *r* or for correct description of an incorrect value of *r*. **Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the solution **Award 2** marks for the correct solution. **Award 1** mark for substantial progress towards the solution

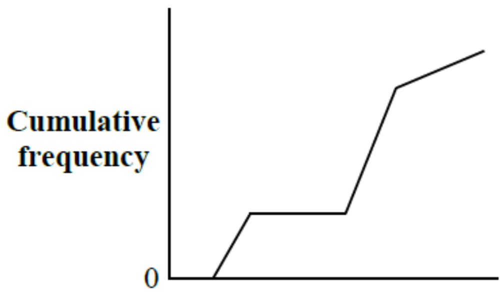
Multiple Choice questions

Question 7: Which of the following CANNOT be a cumulative frequency polygon?

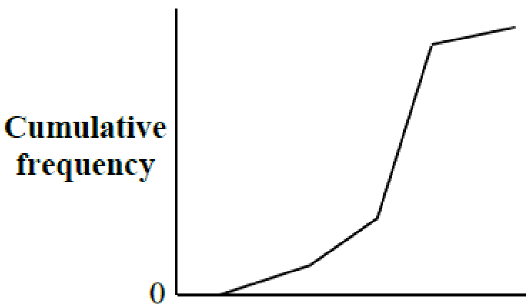
A.



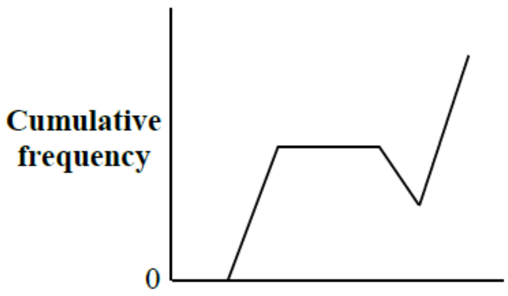
C.



B.



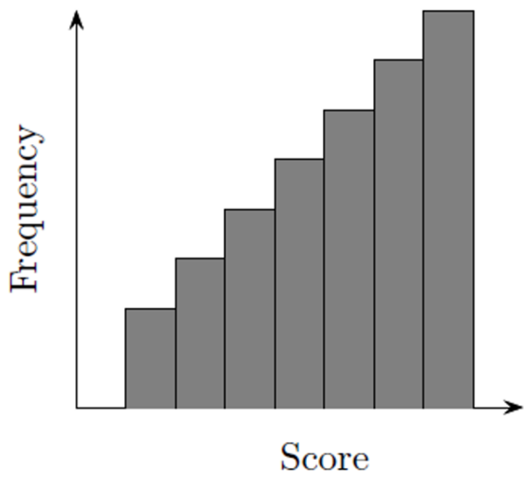
D.



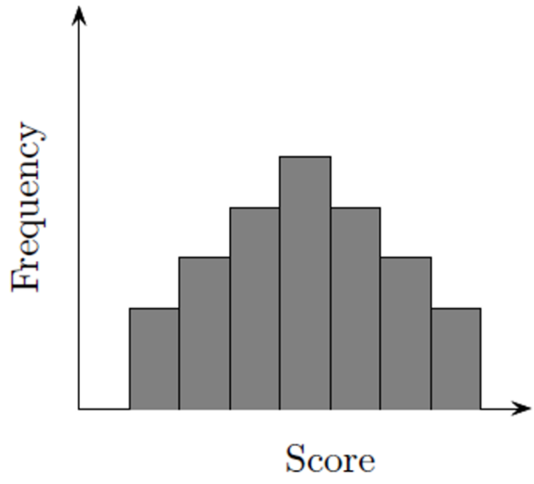
Answer: D . Cumulative frequency is never going to be a decreasing function.

Question 8: Which of the following graphs shows data with the largest standard deviation?

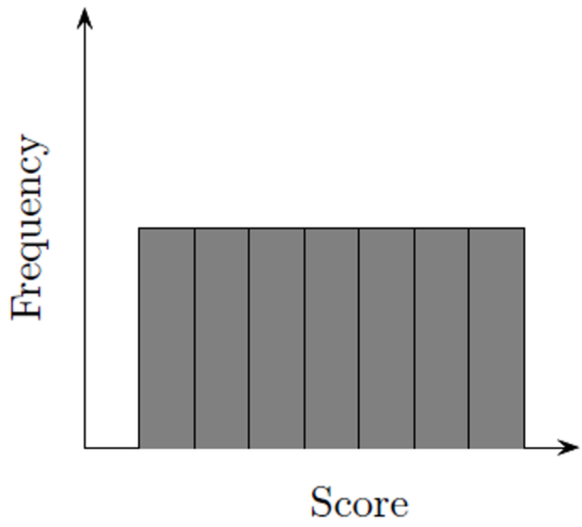
A.



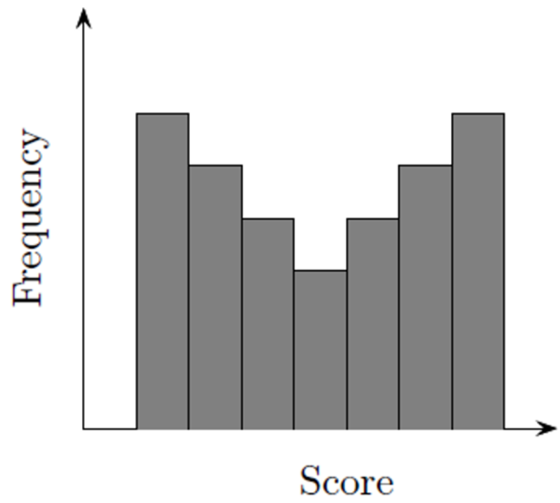
C.



B.



D.



Answer: **D** due to a lot of low scores and high scores the distance between the mean and each of the individual scores will be greater. Hence greater Standard deviation.

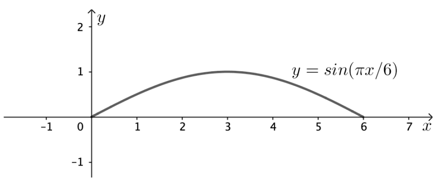
Outcomes Addressed in this Question

MA12-8 solves problems using appropriate statistical processes

Outcome	Solutions	Marking Guidelines
MA12-8	**(a)(i)**	**3 marks** Correct solution with full reasoning/justification **2 marks** Shows value of integral is equal to 1 but neglects to mention $f(x)\geq 0$. OR states $f(x)\geq 0$ with a minor error in integral. **1 mark** Makes some progress towards a correct solution.

$$f(x)=\frac{\pi}{12}\sin\left(\frac{\pi x}{6}\right),\left[0,6\right]$$

For the domain $0\leq x\leq 6$, $\sin\left(\frac{\pi x}{6}\right)\geq 0$ (shown below graphically), hence, $f(x)\geq 0$



Also,

$$\int_0^6 \frac{\pi}{12}\sin\left(\frac{\pi x}{6}\right)dx=\frac{\pi}{12}\left[-\frac{6}{\pi}\cos\left(\frac{\pi x}{6}\right)\right]_0^6$$
$$=\frac{\pi}{12}\left[-\frac{6}{\pi}\cos\left(\frac{\pi\times 6}{6}\right)-\left(-\frac{6}{\pi}\cos\left(\frac{\pi\times 0}{6}\right)\right)\right]$$
$$=\frac{\pi}{12}\left[-\frac{6}{\pi}\cos\pi+\frac{6}{\pi}\cos 0\right]$$
$$=\frac{\pi}{12}\left(-\frac{6}{\pi}\times -1+\frac{6}{\pi}\times 1\right)$$
$$=\frac{\pi}{12}\times\frac{12}{\pi}$$
$$=1$$

$\therefore$ The function is a probability density function since $f(x)\geq 0$ and $\int_a^b f(x)\,dx=1$.

Outcome	Solutions	Marking Guidelines
MA12-8	**(ii)**	**2 marks** Correct solution. **1 mark** Substantial progress towards correct solution.

$$P(X\leq 4)=\int_0^4\frac{\pi}{12}\sin\frac{\pi x}{6}dx$$
$$=\frac{\pi}{12}\left[-\frac{6}{\pi}\cos\frac{\pi x}{6}\right]_0^4$$
$$=\frac{\pi}{12}\left[-\frac{6}{\pi}\cos\frac{\pi\times 4}{6}-\left(-\frac{6}{\pi}\cos\frac{\pi\times 0}{6}\right)\right]$$
$$=\frac{\pi}{12}\left(-\frac{6}{\pi}\cos\frac{2\pi}{3}-\left(-\frac{6}{\pi}\cos 0\right)\right)$$
$$=\frac{\pi}{12}\times-\frac{6}{\pi}\times-\frac{1}{2}+\frac{\pi}{12}\times\frac{6}{\pi}$$
$$=\frac{1}{4}+\frac{1}{2}$$
$$=\frac{3}{4}$$

<table>
<tr><td>MA12-8</td><td>

(b)

$$F(x)=\frac{x^3-8}{335}$$

$Q_3: 0.75=\frac{x^3-8}{335}$ $Q_1: 0.25=\frac{x^3-8}{335}$

$251.25=x^3-8$ $83.75=x^3-8$

$259.25=x^3$ $91.75=x^3$

$x\approx 6.38$ (2 dec. pl.) $x\approx 4.51$ (2 dec. pl.)

$$\begin{aligned}IQR&=Q_3-Q_1\\&=6.38-4.51\\&=1.87\end{aligned}$$

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2 marks
Correct solution, giving correct value for IQR.
1 mark
Substantial progress towards correct solution, showing correct value for one of Q_1 or Q_3.

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<tr><td>MA12-8</td><td>

(c)(i)
Using the empirical law 95% of Barramundi are within 2 standard deviations of the mean, or, 5% are more than 2 standard deviations smaller or larger than the mean. Given the symmetry of the normal distribution, 2.5% of Barramundi are smaller than 2 deviations less than the mean.
Hence,

$$\begin{aligned}54&=\mu-2s\\\mu&=54+2\times 4\\&=62\text{ cm}\end{aligned}$$

ie. Mean length of caught Barramundi was 62 cm.

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2 marks
Correct solution.
1 mark
Substantial progress towards correct solution, showing some knowledge of the empirical law.

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<tr><td>MA12-8</td><td>

(ii)
R minimum length = 30 cm B minimum length = 55 cm

$$z\text{-score}=\frac{x-\mu}{s}=\frac{30-36}{3}=-2 \qquad z\text{-score}=\frac{x-\mu}{s}=\frac{55-62}{4}=-1.75$$

From the z-scores and empirical law, only 2.5% of caught Red Snapper will be returned to the water because they are too small (minimum length is 2 standard deviations from the mean), however, more than 2.5% of Barramundi will be returned to the water as minimum length is only 1.75 standard deviations from the mean. ie. caught Barramundi are more likely to be returned to the water

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3 marks
Correct solution showing z-scores for both species and correct and logical reasoning as to which species is more likely to be thrown back.
2 marks
Two of the three elements correct. Reasoning based on a single incorrect z score is acceptable.
1 mark
One of the three elements correct. Correct reasoning based upon incorrect z-scores is acceptable.

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<tr><td>MA12-8</td><td>

(d)

	second decimal place									
z	+.00	+.01	+.02	+.03	+.04	+.05	+.06	+.07	+.08	+.09
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015

From the table, the z-score for an applicant that is faster than 89.97% of the other applicants is 1.28. The z-score of an applicant that is faster than 90.15% of the other applicants is 1.29.
This mean qualifying time for next stage would be:

$x=\mu-1.28s$ = 5 minutes -1.28×30 seconds = 4 minutes 21.6 seconds

OR

$x=\mu-1.29s$ = 5 minutes -1.29×30 seconds = 4 minutes 21.3 seconds

(Either answer would be acceptable. 1.28 is closer to 90% probability, 1.29 ensures probability exceeds 90%)

Darcy's time of 4 minutes 23 seconds does not qualify her for the next stage of selection.

(This could also be justified by calculating Darcy's z-score. 37 s faster than the mean
gives z = 1.23, less than the 1.28 required.)

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3 marks
Correct solution showing the z-score and time required to progress to next stage and correct conclusion.
2 marks
Substantial progress towards a correct solution with one of the above elements incorrect.
1 mark
Some progress towards correct solution with one of the elements correct.

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